UITWERKINGEN TENTAMEN WISKUNDE 3 JUNI 1993

1 (a) (i) $(TSx, TSy) = (Sx, Sy)$ omdat T unitair is
$= (x, y)$ " S "

(ii) $(Sx, Sy) = (TSx, TSy)$ omdat T unitair is
$= (x, y)$ omdat TS unitair is

(b) $A = \begin{pmatrix} 1 & 3 & 4 \\ 3 & 1 & 0 \\ 4 & 0 & 1 \end{pmatrix}$, $\det(\lambda I - A) = (\lambda - 1)(\lambda - 6)(\lambda + 4)$

$XAX^t = Y \operatorname{diag}(1, 6, -4) Y^t$ met $Y = XC$

eenbladige hyperboloïde

2 a. $Dh = Df(g(s)) \cdot g(s) = \left(\frac{\partial f}{\partial x} \;\; \frac{\partial f}{\partial y} \;\; \frac{\partial f}{\partial z}\right) \begin{pmatrix} \frac{dg_1}{ds} \\ \frac{dg_2}{ds} \\ \frac{dg_3}{ds} \end{pmatrix}$

$$\frac{dh}{ds} = \frac{\partial f}{\partial x} 1 + \frac{\partial f}{\partial y} \cos + 0. = 2x + 2y \cos$$
$$= 2s + 2 \sin s \cos s$$

$h(s) = s^2 + (\sin s)^2 + 1 \Rightarrow \frac{dh}{ds} = 2s + 2\sin s \cos s$

b. $\nabla \underline{l} = \nabla \underline{g} \, \nabla f = \begin{pmatrix} \frac{dg_1}{ds} \\ \frac{dg_2}{ds} \\ \frac{dg_3}{ds} \end{pmatrix} \left(\frac{\partial f}{\partial x} \;\; \frac{\partial f}{\partial y} \;\; \frac{\partial f}{\partial z}\right)$

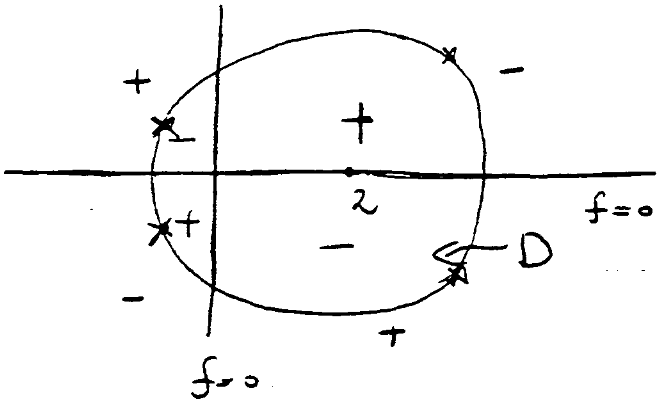
$$\begin{pmatrix} \frac{\partial l_1}{\partial x} & \frac{\partial l_1}{\partial y} & \frac{\partial l_1}{\partial z} \\ \frac{\partial l_2}{\partial x} & \frac{\partial l_2}{\partial y} & \frac{\partial l_2}{\partial z} \\ \frac{\partial l_3}{\partial x} & \frac{\partial l_3}{\partial y} & \frac{\partial l_3}{\partial z} \end{pmatrix} = \begin{pmatrix} 1 \\ \cos \\ 0 \end{pmatrix} (2x \;\; 2y \;\; 2z) = \begin{pmatrix} 2x & 2y & 2z \\ 2x\cos(s) & 2y\cos & 2z\cos \\ 0 & 0 & 0 \end{pmatrix}$$

met $s = x^2 + y^2 + z^2$

$\underline{l}(x, y, z) = \begin{pmatrix} x^2 + y^2 + z^2 \\ \sin(x^2 + y^2 + z^2) \\ 1 \end{pmatrix} \Rightarrow$ het zelfde antwoord

Zie de uitwerking van # 1,2 Toets 5.

4. (a)



tekenschets van $f(x,y)=xy$

f verandert van teken 4 keer binnen D $\Rightarrow$ 4 lokale extrema

(b) $\frac{\partial f}{\partial x} = y = 0$ en $\frac{\partial f}{\partial y} = x = 0 \Rightarrow (0,0)$ Zadelpunt.
Dus geen extrema binnen D

(c) Op ∂D beschouw $xy + \lambda\left((x-2)^2 + y^2 - 12\right)$

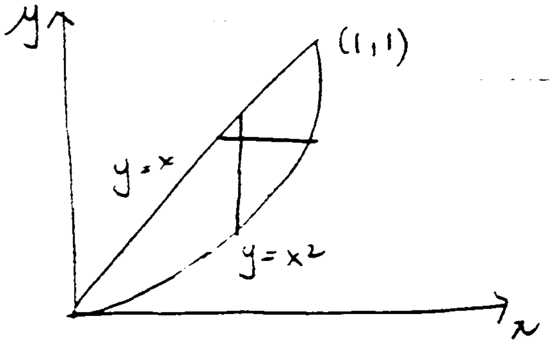
$\left.\begin{array}{ll} \frac{\partial}{\partial x}: & y + 2\lambda(x-2) = 0 \\ \frac{\partial}{\partial y}: & x + 2\lambda y = 0 \end{array}\right\} \Rightarrow$ punten $(4, \pm 2\sqrt{2})$, $(-1, \pm\sqrt{3})$

$f(4, 2\sqrt{2}) = 8\sqrt{2}$, $f(4,-2\sqrt{2}) = -8\sqrt{2}$, $f(-1,-\sqrt{3}) = \sqrt{3}$, $f(-1,\sqrt{3}) = -\sqrt{3}$.

f is continu en D is gesloten en begrensd (∂D ook) $\Rightarrow f$ heeft globaal max en min op $\partial D \Rightarrow$ max in $(4, 2\sqrt{2})$
(en dus ook lokaal max en min in D) min in $(4, -2\sqrt{2})$
Onderzoek $(-1,-\sqrt{3})$ via $f(-1+\varepsilon, -\sqrt{3}+\mu) = \sqrt{3} - \sqrt{3}\varepsilon - \mu + \varepsilon\mu < \sqrt{3} \Rightarrow$ lokaal max
en $(-1, \sqrt{3})$ lokaal minimum.

5 (a) $\int_0^1 \left(\int_{x^2}^{x} x \sqrt{}\, dy\right) dx \Rightarrow$ integratiegebied

$S = \{(x,y) : 0 \le x \le 1 ; x^2 \le y \le x\}$



$= \{(x,y) : 0 \le \le 1 ; \le x \le \sqrt{y}\}$

Dus

$$I = \int_0^1 \left(\int_y^{\sqrt{y}} x \sqrt{\tfrac{1}{2}y^2 - \tfrac{1}{3}y^3}\, dx\right) dy$$

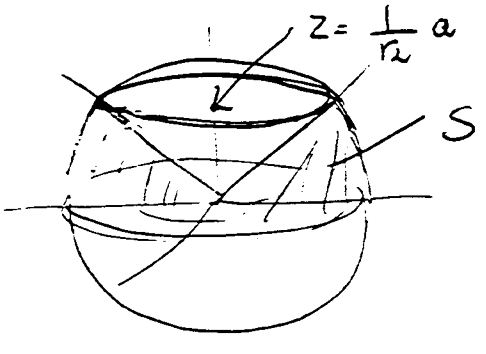
5 a $I = \int_0^1 \left(\sqrt{\tfrac{1}{2}y^2 - \tfrac{1}{3}y^3} \quad \left[\tfrac{1}{2} z^2 \right]_y^{\sqrt{y}} \right) dy$

$= \int_0^1 \sqrt{\tfrac{1}{2}y^2 - \tfrac{1}{3}y^3} \ \tfrac{1}{2} (y - y^2)\, dy$

zet $u = \tfrac{1}{2}y^2 - \tfrac{1}{3}y^3 \Rightarrow du = (y - y^2)\, dy$

$I = \int \tfrac{1}{2}\sqrt{u}\ du = \left[\tfrac{2}{3} \times \tfrac{1}{2} u^{3/2} \right] = \tfrac{1}{3} \left[\left(\tfrac{1}{2}y^2 - \tfrac{1}{3}y^3 \right)^{3/2} \right]_0^1$

$= \tfrac{1}{3} \left(\tfrac{1}{6} \right)^{3/2} = \tfrac{1}{18} \tfrac{1}{\sqrt{6}} = \tfrac{1}{108} \sqrt{6}$.

b. 

halve bol met een kegel uitgesneden

in cylinder coord. (r, θ, z)

$x = r\cos\theta, \quad y = r\sin\theta, \quad z = z.$

$$\begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} & \frac{\partial x}{\partial z} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} & \frac{\partial y}{\partial z} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial z} \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta & 0 \\ \sin\theta & r\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = r$$

$S = \{ (r, \theta, z) \mid z \geq 0,\ r^2 + z^2 \leq a^2,\ r^2 \geq z^2,\ 0 \leq \theta \leq 2\pi \}$

$= \{ (r, \theta, z) \mid z \geq 0,\ z^2 \leq r^2 \leq a^2 - z^2,\ 0 \leq z \leq \tfrac{1}{\sqrt{2}} a,\ 0 \leq \theta \leq 2\pi \}$

$I = \int_0^{2\pi} \left(\int_0^{\frac{1}{2}a\sqrt{2}} \int_z^{\sqrt{a^2 - z^2}} z\, r\, dr \right) dz \Big) d\theta$

$= 2\pi \int_0^{\frac{1}{2}a\sqrt{2}} z\, \tfrac{1}{2} (a^2 - z^2 - z^2)\, dz$

$= 2\pi \left[\tfrac{1}{4} a^2 z^2 - \tfrac{1}{4} z^4 \right]_0^{\frac{1}{2}a\sqrt{2}}$

$= \tfrac{1}{8} a^4 \pi$